

# Fairness for Robust Learning to Rank

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## Summary

- Conventional ranking systems: Maximize the utility of the ranked items to users.
- Fairness-aware ranking systems:** Maximize the utility + balance the exposure for different protected attributes such as gender or race.
- FairRobust LTR** : Fairness-aware + Distributionally Robust
- We derive a new ranking system based on the first principles of **distributional robustness**.
- Provide better utility for highly fair rankings than existing baseline methods.

## Background and Notation

### Ranking Problem

- Rank a candidate set of items  $D = \{d_1, d_2, \dots, d_N\}$
- Ranking/Permutation  $\pi$
- Permutation  $\pi$  places item  $d_i$  at rank  $j$   $\pi_i = j$
- Relevance judgment for  $d_i$   $rel_i$

### Utility of Ranking [general form]:

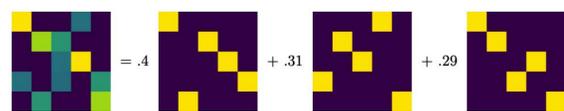
$$U(\pi) = \sum_{d_i \in D} u(d_i) v(j)$$

- $u(d_i)$ : utility of a single item  $d_i$
- $v(j)$ : attention  $d_i$  gets by being placed at rank  $j$  by permutation  $\pi$

### DCG: common evaluation measure in ranking system

$$DCG(\pi) = \sum_{d_i \in D} \frac{2^{rel(d_i)} - 1}{\log(1 + j)}$$

- Searching the space of all rankings is **Exponential in  $N$**
- Solution:** use doubly stochastic matrix  $\mathbf{P} \in \mathbb{R}^{N \times N}$  instead of permutation  $\pi$ .
- $P_{j,k}$ : Probability that  $\pi_j = k$



- Expected utility for a probabilistic ranking in a vectorized form:  $U(\mathbf{P}) = \mathbf{u}^T \mathbf{P} \mathbf{v}$

## Fairness of Exposure in Ranking

- Fairness notions used in probabilistic ranking:  $\mathbf{f}^T \mathbf{P} \mathbf{v} = h$  [1].
- $\mathbf{f}$ : a vector to encode group membership and/or relevance of each document.  $h$  is scalar.

## Distributionally Robust Learning to Rank

- The fair ranking optimization can be expressed as a linear programming problem (post-processing) [1]:

$$\max_{\mathbf{P} \in \Delta \cap \Gamma^{fair}} \mathbf{u}^T \mathbf{P} \mathbf{v} \text{ where: } \Delta : \mathbf{P} \mathbf{1} = \mathbf{P}^T \mathbf{1}, \\ \mathbf{P}_{j,k} \geq 0, \forall 1 \leq j, k \leq M$$

### Extension: Derive an LTR approach. [in-processing]

- Learn to optimize utility and fairness simultaneously.

**Definition:** The fair probabilistic ranking  $\mathbf{P} \in \mathbb{R}^{M \times M}$  in adversarial LTR learns a fair ranking that maximizes the worst-case ranking utility approximated by an adversary  $\mathbf{q}(\tilde{\mathbf{u}})$ , constrained to match the feature statistics of the training data:

$$\max_{\mathbf{P} \in \Delta \cap \Gamma^{fair}} \min_{\mathbf{q}} \mathbb{E}_{\mathbf{X} \sim \tilde{\mathcal{P}}} [U(\mathbf{X}, \mathbf{P}, \mathbf{q})] \\ \text{s. t. } \mathbb{E}_{\mathbf{X} \sim \tilde{\mathcal{P}}} \left[ \sum_{j=1}^M \mathbb{E}_{\tilde{\mathbf{u}}_j | \mathbf{X} \sim \mathbf{q}} [\tilde{\mathbf{u}}_j \mathbf{X}_{j,:}] \right] = \mathbb{E}_{\mathbf{X}, \mathbf{u} \sim \tilde{\mathcal{P}}} \left[ \sum_{j=1}^M u_j \mathbf{X}_{j,:} \right]$$

## Optimization

- Solve the formulation in Lagrangian dual form:

$$\max_{\theta} \mathbb{E}_{\mathbf{X}, \mathbf{u} \sim \tilde{\mathcal{P}}} \left[ \max_{\mathbf{P} \in \Delta} \min_{0 \leq \mathbf{q} \leq \mathbf{1}} \mathbf{q}^T \mathbf{P} \mathbf{v} + \left\langle \mathbf{q} - \mathbf{u}, \sum_l \theta_l \mathbf{X}_{:,l} \right\rangle + \lambda \mathbf{f}^T \mathbf{P} \mathbf{v} \right]$$

- Optimize the dual parameters  $\theta \in \mathbb{R}^{L \times 1}$  for the feature matching constraint of  $L$  features by gradient decent.
- Use  $\lambda$  as a penalty parameter for fairness constraint.

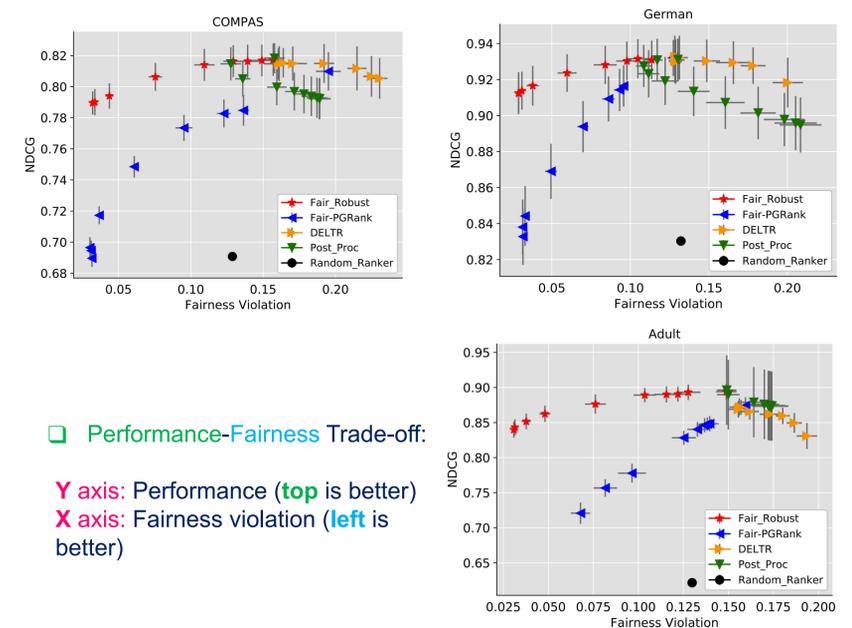
- $\mathbf{u} \in \mathbb{R}^{M \times 1}$  is a vector of true labels.
- $\mathbf{q} \in \mathbb{R}^{M \times 1}$  is a probability vector of the adversary's estimation of each item being relevant.
- $\mathbf{X}_{:,l} \in \mathbb{R}^{M \times 1}$  denotes the  $l_{th}$  feature of  $M$  samples.
- $S$  is the set of protected attributes
- $\mathbf{v} \in \mathbb{R}^{M \times 1}$  is a vector containing the values of position bias function for each position.

**Ranker player[P]:** Choosing a distribution over rankings constrained to provide fairness while maximizing utility.

**Adversary player[q]:** Choosing a distribution of item relevancies that minimizes utility while being similar to training data properties.

## Experiments

- Construct LTR dataset from fair classification datasets.
- Goal:** provide **low fairness violation** and **high NDCG**
- Fair\_Robust** achieves preferable utility-fairness trade-off



### Performance-Fairness Trade-off:

**Y axis:** Performance (top is better)  
**X axis:** Fairness violation (left is better)

- Fairest results achieved by **Fair-Robust** and baseline methods.

Method	COMPAS		German	
	NDCG	$\hat{D}_{group}$	NDCG	$\hat{D}_{group}$
FAIR_ROBUST	<b>0.789 ± 0.007</b>	0.032 ± 0.001	<b>0.912 ± 0.011</b>	<b>0.029 ± 0.002</b>
FAIR_PGRANK	0.696 ± 0.006	<b>0.030 ± 0.001</b>	0.838 ± 0.015	0.031 ± 0.001
DELTR	0.815 ± 0.010	0.160 ± 0.009	0.933 ± 0.011	0.128 ± 0.003
POST_PROG	0.818 ± 0.009	0.158 ± 0.007	0.927 ± 0.011	0.108 ± 0.005

## References

[1] Singh, Ashudeep, and Thorsten Joachims. "Fairness of exposure in rankings." In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pp. 2219-2228. 2018.